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# Integration of conceptual mathematical relationships into constant time delay instruction

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INTEGRATION OF CONCEPTUAL MATHEMATICAL RELATIONSHIPS INTO  
CONSTANT TIME DELAY INSTRUCTION

A Thesis  
Submitted to the Graduate Faculty of the  
Louisiana State University and  
Agricultural and Mechanical College  
in partial fulfillment of the  
requirements for the degree of  
Master of Arts

In

The Department of Psychology

by  
Bethany A. Porter  
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## Abstract

There is constant debate over mathematics education in the United States. One central controversy is whether or not the current methods used to teach students mathematics are effective. Some scholars believe that students are not getting enough practice and that they are not getting a good conceptual understanding of mathematics. It has been shown that mathematics equations are rich in patterns and inter-relationships and when children understand these relationships they have higher mathematic skill levels than their peers who do not. This study examined the effectiveness of using an empirically supported, fast paced mastery oriented teaching procedure that promotes automaticity and fluency while also addressing a conceptual understanding of mathematics. Participants were general education students in the first grade referred for mathematics assistance. In Experiment one, a constant time delay procedure was used to teach the students a set number of single digit addition and subtraction facts with integrated mathematical relationships. In Experiment two, constant time delay was used again to teach the participants 18 single digit addition problems. Effectiveness and efficiency of teaching conditions were evaluated. The results of experiment one are inconclusive with only one participant being able to continue through the whole experiment and little change in her data. However, experiment two demonstrated that the children were able to learn simple addition facts using a constant time delay procedure.

## Introduction

There has been debate over mathematics education in the United States for many years now. Some of the many issues under constant debate include whether there is too much or too little arithmetic being taught, should calculators be used in the classroom, problem solving versus calculations, and are the mathematics teachers qualified enough (Lewis, 2005). Another source of controversy arises from the results posted by the National Assessment of Educational Progress (NAEP). Some scholars focus on the results stating that the scores of American students have risen in the past years and have an optimistic outlook (Loveless & Coughlan, 2004). There are still others who have done their own studies and claim that the results of the NAEP do not mean much because the tests are given at skill levels below the children's actual grade levels (Lewis, 2005). There are also debates over the national standards for mathematics curriculum in the United States. These standards were created by the National Council of Teachers of Mathematics in 1989 (Loveless & Coughlan, 2004). The standards created by NCTM have created controversy within schools. For example, fact mastery is considered important by most teachers, but the ways in which it can be taught that will comply with NCTM standards is unclear (Isaacs & Carroll, 1999).

One of the central questions in the current mathematics controversy is: Are current methods of teaching mathematics effective? Some answer no. Only an estimated 50% of high school juniors and seniors mastered eighth-grade mathematics during the 1989-90 school year (Engelmann et al., 1991). The Trends in International Mathematics and Science Study (TIMSS) 2003 results showed that U.S. fourth graders performed 12<sup>th</sup> of 25 nations (Lewis, 2004). Various scholars and policy makers have argued that United

States students should be number one in line with the Goals 2000: Educate America Act (Maccini & Gagnon, 2000). One of the reasons many feel that this is not accomplished is because U.S. students do not learn the fundamentals (Lewis, 2004).

Fundamentals, such as basic fact knowledge, are the stepping stones for students in mathematics (Koscinski & Gast, 1993). Computation skills are necessary to advance in mathematics and the sciences, they are an increasingly important predictor of adult earnings, and they promote equity in mathematics achievement (Loveless & Coughlan, 2004). The focus of the current study is the basic mathematical skill of computation of addition and subtraction problems.

#### Mathematical Instruction Literature Review

Most U.S. curriculums represent a spiral curriculum where certain topics are introduced in one grade and then repeated with more detail in a later grade (Zhou & Peverly, 2005). This curriculum causes different problems. One problem is the result of students being restricted the opportunities for learning and practice. Porter (1989) reported that 70% or more of the topics covered received less than 30 minutes of instruction time. Another problem could result from the instruction lacking in depth and coherence. One analysis of a currently popular mathematics series revealed that 76% of the material in Grade 6 is review, 80% in Grade 7, and 82% in Grade 8 (Engelmann et al., 1991).

Engelmann et al. (1991) reports that research conducted at the University of Oregon has identified at least six additional factors that might contribute to the poor performance of U.S. students: 1) provisions to ensure that the student has prior relevant knowledge are often marginal; 2) the rate at which new concepts are introduced is often



too fast; 3) the presentation of strategies often lacks coherence; 4) many instructional activities are not communicated in a clear, concise manner; 5) the transition, in the form of guided practice, between the initial teaching stage and the stage in which students work independently is usually inadequate; 6) the review provided to ensure that students will remember what they have learned is at times sparse, or absent entirely.

Most research on mathematical instruction focuses on problem solving, but fundamentals of mathematics, like addition and subtraction, need to be investigated because they are essential for students to proceed with success in mathematics. Most children learn addition with little or no problem. It is when subtraction is introduced that problems more frequently arise. In one study, Putman, deBettencourt, and Leinhardt (1990) investigated children's justifications and evaluations of derived-fact strategies- strategies for solving addition and subtraction problems by using known combinations. They found that about half of the students were able to give adequate explanations of the derived-fact strategies for addition, but only about 10-20% of the students were able to give adequate explanations for subtraction strategies. This could be because children have a harder time with subtraction and are less fluent with their knowledge of subtraction, or that they simply don't get enough practice.

Arithmetic equations have a rich pattern of structure and inter-relationships. For instance, half of the addition combinations are related to one another by the commutative law. Addition and subtraction are complementary processes. When children learn computation procedures they have the opportunity to observe and learn these structures and relationships (Baroody et al., 1983). This structure can facilitate computation by eliminating unnecessary effort. For example, by giving a child the combination of  $7+3$

and  $3+7$ , using the commutative principle, only one calculation need be made. The same can be used with subtraction and the complementary principle. This use of mathematical structure can lead to efficient and intelligent problem solving. Baroody et al. (1983) examined the use of the commutative, addition-subtraction complement, and  $N+1$  progression principles in solving number combinations by capable first-third graders. Concerning only the addition-subtraction complement principle, a significant association was found between grade and the first use of the principle. All third graders used the complement principle on the first trial, whereas only a few first and second graders did. On the second trial though, more first and second graders used the principle than in the first trial. It appeared that the use of the complement principle seemed to be associated with an efficient solution of the related problem (p. 163). So, more practice and a better understanding of the addition problem may lead to a quicker gain of knowledge of the complement fact. To be more efficient with addition problems, one solution may be more practice with the problem and also using the commutative principle ( $3+4=7$  and  $4+3=7$ ). In this same Baroody study, he found that the commutative principle is well known by most young children.

As mentioned earlier, a spiral curriculum is generally used in most general education classrooms. This means that addition is introduced at some point, and then at a later point, subtraction is taught. This process does not give the child the opportunity to make connections between the two functions. Some have argued that addition and subtraction should be kept separate until the child can answer both types of problems with ease (Cordoni, 1987). However, by not allowing the children to see both applications of addition and subtraction at the same time, no connections can be made

about the structures relating to the mathematical principles. It has been recognized that having the knowledge and being able to use mathematical structure can lead to more efficient and intelligent problem solving (Wertheimer, 1945). Unfortunately, very little research has been done to demonstrate that introducing addition and subtraction at the same time is either harmful or beneficial to the student. It has been thought though that the knowledge of addition can aid in the learning of subtraction processes (Baroody, 1999).

Baroody (1999) investigated children's knowledge of the complementary principle between addition and subtraction. He found that performance on a complement task appeared to be related to prior experience with subtraction. He concluded that the complement principle between addition and subtraction was not readily recognized by the children in his study. However, once children were exposed to the complement principle they were able to recognize it and completed tasks much faster than when the task was first introduced. This finding may suggest that if addition and subtraction are presented simultaneously, the complement principle would be more obvious and lead children to more efficient addition and subtraction skills in a shorter period of time.

Learning the basic fundamentals of mathematics, such as addition and subtraction, is a vital aspect of a student's future success. Mathematics is essential for all students to succeed. The fact is, though, that many students have problems with mathematics. Some of these problems may be explained when looking at comparisons of activities across countries (Maccini, 2000). For example, U.S. students spend 96% of their seatwork time on routine procedures, whereas Japanese students engage in this type of activity approximately 41% of the time. Rarely are U.S. students required to work on activities

where they have to invent new solutions, proofs, or creative procedures (Maccini).

Teaching students using a spiral curriculum does not allow the student to get in depth, thorough instruction on a topic. This in turn leads to the students not being able to make connections between structure and mathematical principles. The National Council of Teachers of Mathematics (NCTM) has proposed new curricular, assessment, and teaching practices that emphasize complex mathematical tasks requiring problem solving and mathematical reasoning skills and deemphasizes rote computation and memorization tasks (Maccini, p. 1). The problem with this, however, is getting teachers to change their way of teaching to incorporate this new curricula.

#### Constant Time Delay Literature Review

One major key to the development of mathematical skills for students is the teaching method. Traditional rote methods include the use of frequent drill and timed tests which have disadvantages (Isaacs & Carroll, 1999). These methods may induce anxiety and lead the students to a misunderstanding of the concept (Isaacs & Carroll). Other more efficient strategies for teaching students, including those with learning disabilities, include constant and progressive time delay (Handen & Zane, 1987), the system of least prompts (Doyle, Wolery, Ault, & Gast, 1988), most-to-least prompting (Billingsley & Romer, 1983), and integrated prompting procedures and stimulus shaping and fading (Ault, Wolery, Doyle, & Gast, 1989; Shoen, 1986). All of these procedures have substantial literature bases documenting their effectiveness.

Morton and Flynt (1997) compared constant time delay and prompt fading to teach multiplication facts to students with learning disabilities. The main purpose of the study was to compare the effectiveness of the two prompting techniques. As described

by Wolery, Ault, and Doyle (1992) constant time delay is a constant interval of time between the presentation of a natural stimulus and a prompt stimulus. They also describe prompt fading as highlighting a physical dimension of a stimulus to increase the likelihood of a correct response and gradually diminishing the prompt over time. Both of these procedures were shown to be effective. However, the constant time delay procedure produced a higher acquisition and retention of the facts for more of the students than did the prompt fading. These results need to be interpreted with some caution. Differences in acquisition and retention may have resulted from the more active role the experimenter played with the constant time delay procedure. Prompting procedures which involved highly predictable answers may have been tiring for the students as compared to the constant time delay.

Studies have compared the efficiency of constant time delay to other response prompting procedures on measures such as the number of trials, errors, and minutes of direct instruction time to criterion (Doyle & Gast, 1990). Constant time delay has been shown to be more efficient than other prompting procedures such as the system of least prompts (Doyle, Wolery, Gast, & Ault, 1990) and just as efficient as progressive time delay (Ault et al., 1988). Not only is constant time delay efficient, it is also easy for teachers to implement and monitor (Stevens & Schuster, 1988). Constant time delay has also been shown to result in a low rate of error during instruction.

Constant time delay procedure is a systematic procedure to facilitate near error free learning by providing models of correct responses until the student responds independently without the model (Schuster et al., 1990, p. 307). There are two different types of trials in constant time delay procedures. In the first trial the student is

immediately given the model following the prompt. After a student has had the chance to respond immediately following the model, leaving little chance for error, the student is then given the opportunity to respond without immediate presentation of the model. A predetermined amount of time is allowed to elapse between the prompt and the model to allow the student a chance to respond. The student is taught to wait for the correct response when they are not able to give the correct response. This time delay is kept constant the remainder of the procedure until criterion has been reached. Since the procedure includes reinforcement for correct responses, either before or after the prompt, a high level of success and low error rate are typically experienced (Schuster, p. 307).

Five types of responses are possible in this procedure, two correct responses and three types of error. Correct nonwait answers are those that are initiated by the student within three seconds, prior to the experimenter giving the model. Correct wait answers are those answers that are correct after the experimenter has given the model. The first error response, nonwait error, occurs when the student answers incorrectly within the three seconds, prior to the experimenter giving the model. The second error is a wait error and is defined as when a student waits for the model and then answers incorrectly. The final error, no response, occurs when the student fails to give any response within the three seconds, and after a model has been given (Cybriwsky & Schuster, 1990).

Constant time delay instruction has been used in many research investigations and has been found to be efficient and effective. Cybriwsky and Schuster (1990) used constant time delay to teach multiplication facts to an elementary-age student with both mild learning disabilities and behavioral disorders. Using a 4 s constant time delay procedure, they were effective in teaching the student 15 multiplication facts in

approximately one hour of instruction. They reported that the implementation was easy, required little preparation, and resulted in a low rate of error (Cybriwsky & Schuster).

Another study by Wolery, Cybriwsky, Gast, and Gast (1991), examined the effectiveness of constant time delay in teaching adolescents with learning or behavior disorders. Wolery et al. taught five students social studies and health facts in a small group format. All students learned the targeted facts with little error. They found that the constant time delay procedure was effective and reliable, even in small group design.

Overall, constant time delay has been proven to be an efficient and effective method of teaching a wide array of students and skills. The evidence supporting the use of constant time delay is why this procedure has been chosen for the current study.

#### Rationale and Purpose of Current Study

The purpose of this study was to examine the effects of bridging two domains thought to be in conflict. These are teaching conceptually rich, deep and professionally recommended understanding of mathematics with an empirically supported, fast paced mastery oriented teaching procedure that promotes automaticity and fluency. By doing so, students can benefit by learning the mathematical relationships in an efficient, coherent manner.

## Experiment I Method

### Participants and Setting

Five first grade general education students (Kate, Sun, Claire, Jack, & John) attending an East Baton Rouge Parish Public School were selected to participate in this study. All students were referred by their teachers for poor performance in mathematics. Parental consent was obtained for all students participating in the study. The students were first screened to make sure they could read numbers correctly and add and subtract with manipulatives (see below). The students were then assessed using a set of 90 single digit addition and subtraction flashcards (45 addition and 45 subtraction) using a constant time delay procedure (see below). The students were not receiving additional services from the school in the area of mathematics, enrolled in resource classes, or taking medications for behavioral problems or learning disabilities.

All sessions were conducted in a quiet room available in the student's school. All rooms contained chairs and a table/desk for the student and consultant to work. This setting was usually an empty room, the library, or the cafeteria. Times when the sessions were conducted were determined by the teacher.

### Screening and Materials

Following teacher referrals, all participants completed a pre-intervention screening. The pre-intervention screening assessed two student skills: reading numbers, and addition and subtraction using manipulatives. Each student was assessed for both skills on one day. After the screening session, the student was given a tangible reward contingent on compliance with demands.



Screening for number recognition was done with a set of 10 flashcards. The flashcards were 3x5 white cards with black ink. Each card had a number (1-10) written on it. The student was given 3s to identify the number on the card. The cards were reviewed three times each. Each student had to complete this screening with 100% mastery. If a student failed this screening, the experimenter would have worked with the student using constant time delay instruction until 100% mastery was achieved prior to intervention. All five participants accurately read numbers 1 through 10 after the review. No additional instruction was needed.

Next, each student was screened to assess comprehension of addition and subtraction using manipulatives. Ten small blocks were used as the manipulatives. The experimenter showed the student the blocks. The experimenter then proceeded to place a certain number of blocks in one group and another number of blocks in a second group in front of the student. The experimenter then asked the student, "If I add these blocks to this group, how many do I have?" For example, one group of blocks may have had three blocks and the other group two blocks. The experimenter asked how many blocks there would be if this group of blocks was added to that group of blocks. The student was given five seconds to answer the question. The student was allowed to physically move the blocks and count them. The same procedure was done to assess for subtraction comprehension, the only difference being that there was only one group of blocks placed in front of the student and the experimenter asked how many blocks were left a certain number of blocks were taken away. A total of 10 addition and 10 subtraction problems was completed. A student had to complete this screening with 100% mastery.

If a student failed to do this, the experimenter worked with the student using constant time delay instruction until 100% mastery was achieved before the intervention started.

One participant, Claire, received one additional day of constant time delay instruction on adding and subtracting with manipulatives. She achieved mastery within one session. All other participants performed this task with 100% accuracy during screening.

Participants were then screened to determine the addition and subtraction facts that they currently knew. This was done using a constant time delay procedure. A set of 90 flashcards (45 addition and 45 subtraction) was used. Cards were presented to the participant in a random order. The participant was allowed 3 seconds to answer. Each card was only presented once. The cards were then put into piles according to their answer as being either correct or not correct. Answers were then recorded on a data sheet.

Materials for the screening consisted of ten 3x5 white cards with numbers 1 through 10 written in black ink and a set of 10 blocks. Materials for the procedure contained a deck of 90 flashcards, 3x5 white cards with black ink, with single digit addition and subtraction on them (45 addition and 45 subtraction). This deck was used for the initial assessment and also for daily progress monitoring. Three additional groups of flashcards were used for each different condition (Experimental, Control, and Traditional). The mathematical equations in each deck were determined after the initial assessment, but the decks were similar in the number of equations they contained (Experimental had 14, Control had 15, and Traditional had 14). The Experimental Group consisted of addition facts with both addends present and the answer missing (e.g.

4+3=\_\_). It also contained facts with the answer and only one addend (e.g. 4+\_\_=7).

The Control Group contained only addition facts, with both addends and only the answer missing. Finally, the Traditional Group consisted of a number of single digit addition and subtraction facts with only the answers missing. No group contained a fact that was present in any of the other groups. Not only was the fact not duplicated within groups, the facts complementary or commutative facts were also not allowed in other groups. A kitchen timer was used to time each session. Data sheets were used to record correct/not correct responses for each session to track progress. The data sheets were divided to record correct facts that belonged to each fact set. The complementary facts to those facts in the three groups were recorded in the data columns of their respective set (e.g. if in the Experimental Group an equation was  $4+3=$ \_\_,  $4+$ \_\_=7, or \_\_+3=7, its complementary facts of  $7-3=$ \_\_ and  $7-4=$ \_\_ were added to the data column of the Experimental Group and could not belong in any other data column).

A box containing rewards (stickers, pencils, candy, etc.) was used. These rewards were available for the students to pick from contingent on their participation during each session.

### Response Definitions

Five types of student responses were possible. There were two correct responses: correct nonwait and correct wait. The three types of errors were: nonwait, wait, and no response. These responses and their definitions were adapted from Cybriwsky and Schuster (1990). The only response that was recorded was the correct nonwait. A correct nonwait response was defined as the student stating the correct answer within three seconds, before the experimenter's prompt.

## Data Collection

Daily data collection included conducting a progress monitoring test using the 90 fact deck set at the start of each session. The cards were sorted into piles of correct and not correct/no response. The experimenter recorded on the data sheet those facts that were answered correctly. Each fact was arranged in a column according to the condition group in which it belonged (Experimental, Control, or Traditional). The data were then graphed. After the adjustment of the procedure of the experiment, progress monitoring was conducted before and after each instructional session for all participants except for Sun who showed gains with the initial procedure and therefore continued on with it.

## Inter-observer Agreement

A second experimenter was trained in the data collection procedures for the progress monitoring tests. The experimenters independently and simultaneously observed and recorded student's correct responses during the test. IOA was calculated by dividing the total number of agreements by the number of agreements plus disagreements for each session, and multiplying by 100.

IOA was collected on 44.44% of Kate's instructional sessions. Mean IOA was 99.93% (range, 99.72% to 100%). IOA was collected on 38.89% of Sun's instructional sessions. Mean IOA was 99.92% (range, 99.80% to 100%). IOA was collected on 38.46% of Claire's instructional sessions. Mean IOA was 99.58% (range, 97.48 to 100%). IOA was collected on 50% of Jack's instructional sessions. Mean IOA was 100% (range, 100% to 100%). IOA was collected on 40% of John's instructional sessions. Mean IOA was 99.91% (range, 99.65 to 100%).

## Procedure

After the initial assessment, the facts were divided into the three similar sized groups of an Experimental, Control, and Traditional Group. The facts were also used to make up additional fact equations, such as those with missing addends and commutative facts and placed in the appropriate condition group. Sessions started with a progress monitoring test. Then, each fact set was taught individually for five minutes. The order in which the students were taught each set was systematically randomized. Each set was taught using a constant time delay procedure. Sessions lasted approximately 20-25 minutes. A reward was given to each student at the end of the session contingent on their compliance.

Sun participated in this procedure for a total of 18 days. For the other four participants, this procedure was conducted for one week after which it became clear that the number of cards in each set was too high for the students. The students were having difficulty paying attention and learning the facts. Thus, the experiment was adjusted slightly for all participants except for Sun. The number of facts in each set was lowered from eight facts to four facts in each set. This allowed for a smaller number of cards for each set and more practice per fact for each equation. Progress monitoring then was also conducted before and after each instructional session with the complete 90 fact deck.

Progress Monitoring. Progress monitoring started by the experimenter explaining to the student that he or she was to try and answer some facts. This set of facts, a total of 90, had addition and subtraction facts all with only the answer missing. The experimenter first gave the task direction (What is the answer?) and then started presenting the cards. Each card was presented one at a time for a total of 3 seconds. No

feedback was given during the administration of this set. The experimenter sorted the cards during the process into piles of correct or not correct/no response for data collection purposes later. Data was then graphed.

Instructional Sessions. The constant time delay procedure was modeled after the procedure used in the Cybriwsky and Schuster (1990) study with slight adjustments (the time delay was changed from 4 s to 3 s). Sessions started with the experimenter telling the student that he or she should just repeat the answer after it was given. The experimenter then set the timer for five minutes. Teaching started with the fact set that had been randomly assigned. For the first presentation of the cards there was a 0 s interval between the task direction (What is the answer?) and the controlling prompt (verbal model of the answer). That is, the experimenter showed the card and immediately stated the answer. The student repeated the correct answer. The cards were then shuffled. For the second presentation of the cards there was a 3 second delay between task direction and the controlling prompt.

The 3 second delay trials started by the experimenter telling the student that he or she should now state the answer if he/she knew it and if not the answer would be given to them shortly. The experimenter then started holding up the cards, one by one, and waited for the student to answer. If the student answered correctly within 3 seconds the experimenter gave verbal praise and moved on to the next card. If the student failed to answer the equation within the 3 seconds the experimenter gave the controlling prompt (verbal model of the answer) and the student was directed to repeat. If the student answered incorrectly, the experimenter would say, “No, the correct answer is\_\_\_,” and then student repeated the answer. There were five possible responses during instruction:

correct nonwait, correct wait, incorrect nonwait, incorrect wait, and no response. The experimenter gave verbal praise after each correct nonwait and wait response. This procedure was repeated for the remaining two fact sets. The student was given a reward contingent on compliance during the session. The student then returned to his or her classroom.

### Experimental Design

A variation of a multiple element, within subjects design was used to evaluate the effectiveness of each condition. The number of correct responses during progress monitoring was graphed. Correct responses were graphed according to which group the correctly answered fact belonged to: Experimental, Control, or Traditional. The visual inspection of the relationships of the data paths representing each of the three instructional methods was used to make comparisons as to which proved to be the most effective method of teaching addition and subtraction.

## Experiment I Results

### Pre-Intervention Screening

The initial screening to determine that the students knew their numbers showed that all participants could identify numbers 1 through 10. Kate, Sun, Jack, and John were able to complete pre-intervention screening for knowledge of addition and subtraction with manipulatives. Claire did not complete this screening with 100% accuracy and therefore was taught addition and subtraction with manipulatives using a constant time delay procedure. After teaching, Claire was able to complete the screening activities with 100% accuracy the second time. Screening also consisted of identifying facts participants already contained in their repertoire. Each participant knew less than 5 total facts. Kate knew only one fact, Sun knew three facts, Claire knew three facts, Jack and John both knew two facts. Because each participant knew only a minimal number of facts, all addition and subtraction problems were left in their instructional groups.

### Instructional Sessions

Sun was the only participant to continue with Experiment 1 the full length of the study. Sun's results for both addition and subtraction facts learned are depicted in Figure 1. The results show that Sun learned 12 of 14 addition facts from the Experimental Group (86%), 12 of 15 from the Control Group (80%), and 10 of 14 from the Traditional Group (71%). She learned the facts in all groups at a steady pace.

Sun's results for subtraction facts reveal that she learned the most facts from the Experimental Group. She learned a total number of 5 of 14 subtraction facts from the Experimental Group (36%), 1 of 15 facts from the Control Group (7%), and 3 of 14 facts from the Traditional Group (21%). The results show that Sun quickly learned a few facts



by session three and then from session three through eleven her progress remained unchanged. Her progress only slightly improved after session eleven for the Control Group and after session thirteen for the Experimental and Traditional Groups. Her progress did not improve again. For the Experimental Group at session sixteen, her progress improved and looked like it was trending upward at a gradual pace until the end of the study. With such a slow trend and a low amount of subtraction facts learned in all groups, the results for Experiment 1 are inconclusive as to whether introducing subtraction facts with addition facts using the complimentary principle is more efficient and effective than teaching each mathematical skill by itself.

There are no results presented for the other four participants because this procedure was found to be ineffective and too hard for them. Thus, these participants were started on Experiment two.

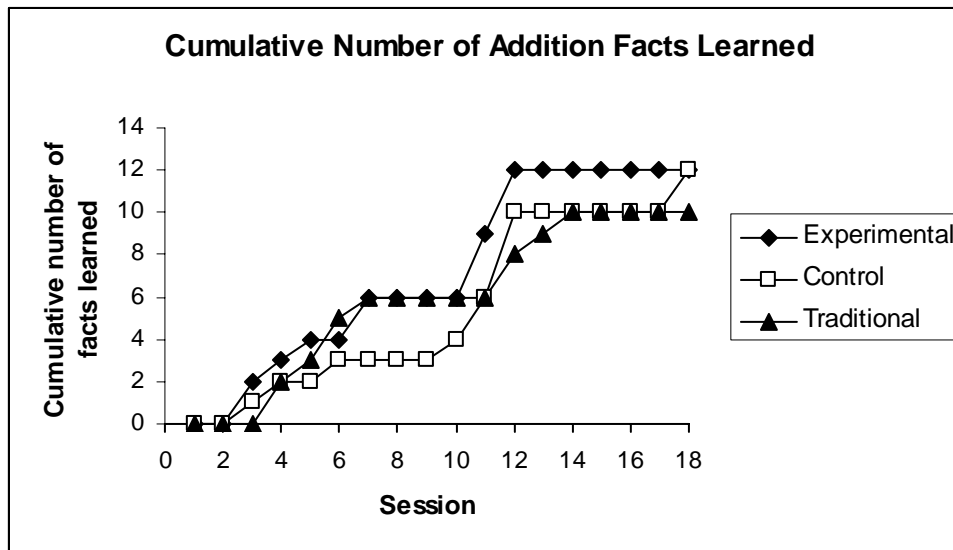
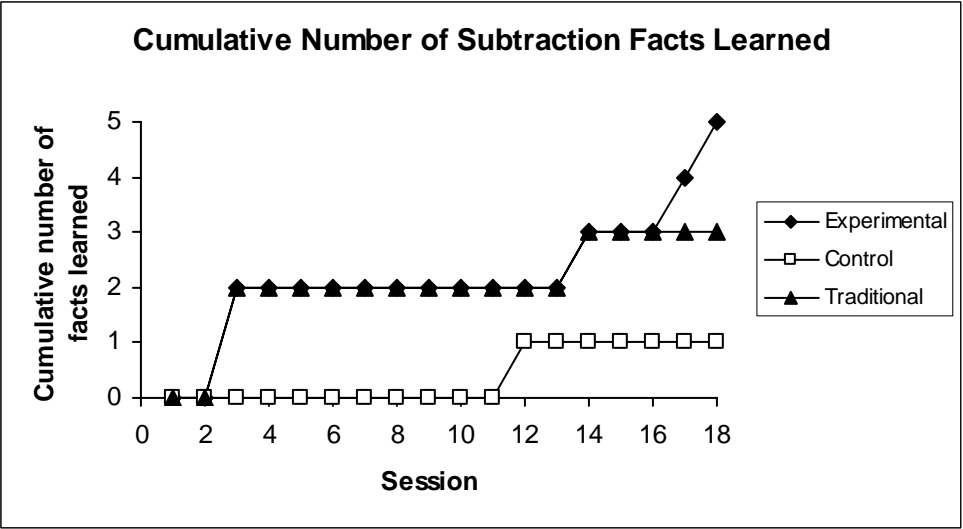


Figure 1 Sun's cumulative number of facts learned for addition and subtraction (fig. con'd).



## Experiment II Method

### Participants and Setting

Four of the five students who participated in Experiment one (Kate, Claire, Jack, & John) participated in Experiment two. Of the four participants, only three finished the study. One participant, John, declined to continue with Experiment two. All sessions were conducted in a quiet room with chairs and a table or desk. This setting was usually an empty room, the library, or the cafeteria within the student's school. Times when the sessions were conducted were determined by the teacher.

### Screening and Materials

Screening for experiment two was conducted in one session. Addition facts that had never been answered correctly were chosen from the groups in experiment one to make up the three sets for experiment two. Screening was done to ensure that the participants did not know these facts. Screening consisted of presenting the facts, one at a time for three seconds, and removing the facts that the participant was able to answer. Once 18 facts were found that the student could not answer correctly the session was stopped. These 18 facts were then divided into equal sets of three and used as the sets A, B, and C. A tangible reward was given for the compliance during the session.

Materials for experiment two consisted of eighteen 3x5 white cards with simple addition facts written in black ink on them. The eighteen facts were pulled from Experiment one. Each fact was chosen based on the participant having never answered the fact correctly in Experiment one. The 18 facts were then randomly split up into three equal sets (A, B, & C) with each group containing a total of six facts. A kitchen timer was used to time each session. Data sheets were used to record correct/not correct

responses for each session to track progress. The data sheets were divided up to record correct facts that belong to each fact set.

A box containing rewards (stickers, pencils, candy, etc.) was used. These rewards were available for the students to pick from contingent on their participation during each session.

### Response Definitions

Five types of student responses were possible. There were two correct responses: correct nonwait and correct wait. The three types of errors were: nonwait, wait, and no response. These responses and their definitions were adapted from Cybriwsky and Schuster (1990). The only response that was recorded was the correct nonwait. A correct nonwait response was defined as the student stating the correct answer within 3 s, before the experimenter's prompt.

### Data Collection

Daily data collection included conducting a progress monitoring test using the 18 fact deck set at the beginning and end of each session. The cards were sorted into piles of correct and not correct/no response. The experimenter recorded those facts that were answered correctly on the data sheet. Each fact was arranged in a column according to the condition set in which it belonged (A, B, or C). The data for the progress monitoring done before the instructional sessions was then graphed. Using the data from the progress monitoring before instructional sessions were conducted required an approximately 24-72 hour retention interval to demonstrate fact mastery.

## Inter-observer Agreement

A second experimenter was trained in the data collection procedures for the progress monitoring tests. The experimenters independently and simultaneously observed and recorded student's correct responses during the test. IOA was calculated by dividing the total number of agreements by the number of agreements plus disagreements for each session, and multiplying by 100.

IOA was collected on 23.08% of Kate's instructional sessions. Mean IOA was 99.30% (range, 96.39% to 100%). IOA was collected on 36.84% of Claire's instructional sessions. Mean IOA was 98.74% (range, 95.54% to 100%). IOA was collected on 31.58% of Jack's instructional sessions. Mean IOA was 99.33% (range, 96.70% to 100%).

## Procedure

After the initial assessment, the facts were divided up into the three sets of six facts (A, B, and C). Sessions started and ended with a progress monitoring test. Each student was taught set A for 5 minutes using a constant time delay procedure. Set A was taught until all six facts had been taught and five out of the six facts met the criteria for having been learned. Facts were defined as learned when the participant answered the fact correctly during progress monitoring three days in a row. One participant, Jack, struggled to learn all the facts and so the requirement of learning five of the six facts was changed to four of six for a set to meet the criteria for having been learned.

Each instructional session for each set began by teaching only one fact. A new fact was added during the instructional sessions when the student was able to answer that fact correctly approximately 10 times in a row. For example, in the first session the timer

was set for five minutes and then one equation from Set A was taught. If the student was able to answer this fact correctly ten times in a row a second fact was added. If the participant could answer both the facts ten times in a row, a third fact from Set A was added and so forth. This procedure was continued until the five minutes was up. Once five of the six facts from Set A met criteria as having been learned, Set B was then taught with the same method.

Kate learned Set A after 8 sessions, Set B after 9 sessions, and Set C after 8 sessions. Claire learned Set A after 4 sessions, Set B after 5 sessions, and Set C after 8 sessions. Jack learned Set A after 12 sessions, Set B after 5 sessions, and Set C after 1 session. John learned Set A after 9 sessions. John did not participate any further in the study after learning Set A.

Progress Monitoring. Progress monitoring started by the experimenter explaining to the student that he or she was to try and answer some facts. This set of facts, a total of 18, contained only simple addition facts. The experimenter first gave the task direction (What is the answer?) and then started presenting the cards. Each card was presented one at a time for a total of 3 seconds. No feedback was given during the administration of this set. The experimenter sorted the cards during the process into piles of correct or not correct/no response for data collection purposes later. Data were then graphed.

Instructional Sessions. The constant time delay procedure was modeled after the procedure used in the Cybriwsky and Schuster (1990) study with slight adjustments (the time delay was changed from 4 s to 3 s). All participants were taught fact set A, then B, and then C. The experimenter would first set the timer for five minutes. The first fact from set A was presented with a 0 s interval between the task direction (What is the

answer?) and the controlling prompt (verbal model of the answer). That is, the experimenter showed the card and immediately stated the answer. The student repeated the correct answer. Next, the fact was presented and the participant was allowed 3 s to answer the fact. If the participant failed to answer or answered incorrectly within the 3 s, the experimenter modeled the correct response. The participant then repeated the answer and was given praise. This fact was presented alone until the participant could correctly answer it approximately 10 times in a row. Once this was achieved, a second fact was added and the process was started over. New facts were added during the 5 minutes of instruction time only if the participant could answer the facts presented approximately 10 times in a row. New facts were added as necessary until the five minutes expired.

Once the participant could correctly answer five out of the six facts (four for Jack) during pre progress monitoring, the next set was introduced. Review of previously learned sets was randomly implemented during subsequent sessions. A set was reviewed on a random schedule of every 3 or 4 sessions. Review consisted of taking all six facts from the review set and using a constant time delay instruction. First was a 0 s delay and then the cards were shuffled. The experimenter then allowed a 3 s delay for the participant to answer. Once all cards had been reviewed the experimenter moved on to the instructional set for that session.

There were five possible responses during instruction: correct nonwait, correct wait, incorrect nonwait, incorrect wait, and no response. The experimenter gave verbal praise after each correct nonwait and wait responses. This procedure was repeated for the remaining two fact sets. The student was given a reward contingent on compliance during the session. The student then returned to his or her classroom.

## Experimental Design

A multiple baseline design was used for experiment two. The number of correct responses given during the pre instructional progress monitoring was graphed. Correct responses were graphed according to which set the correctly answered fact belonged, A, B, or C. The visual inspection of the data paths represent the speed at which each participant learned the addition facts using a constant time delay instruction.



## Experiment II Results

### Baseline

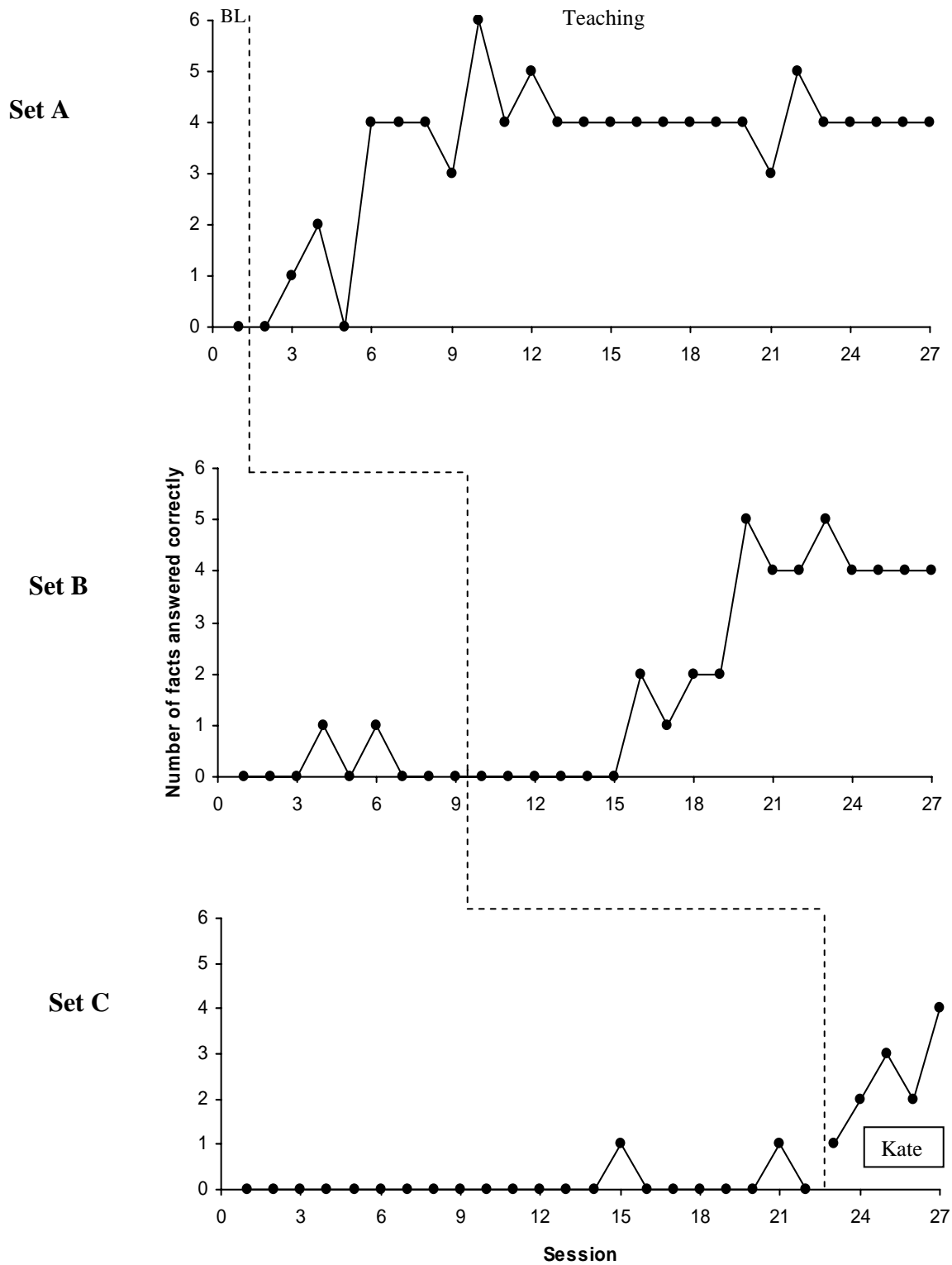
During baseline, participants were not taught facts that did not belong in the current instructional set. So, during the instruction of Set A, facts in Set B and Set C were not taught, but still included in progress monitoring. During baseline, facts were recorded as correct answers if the participant gave the correct answer to the addition fact when it was presented within 3 s. All participants showed minimal gain during baseline for the facts belonging to the sets not being instructed except for set C for Jack.

### Treatment Conditions

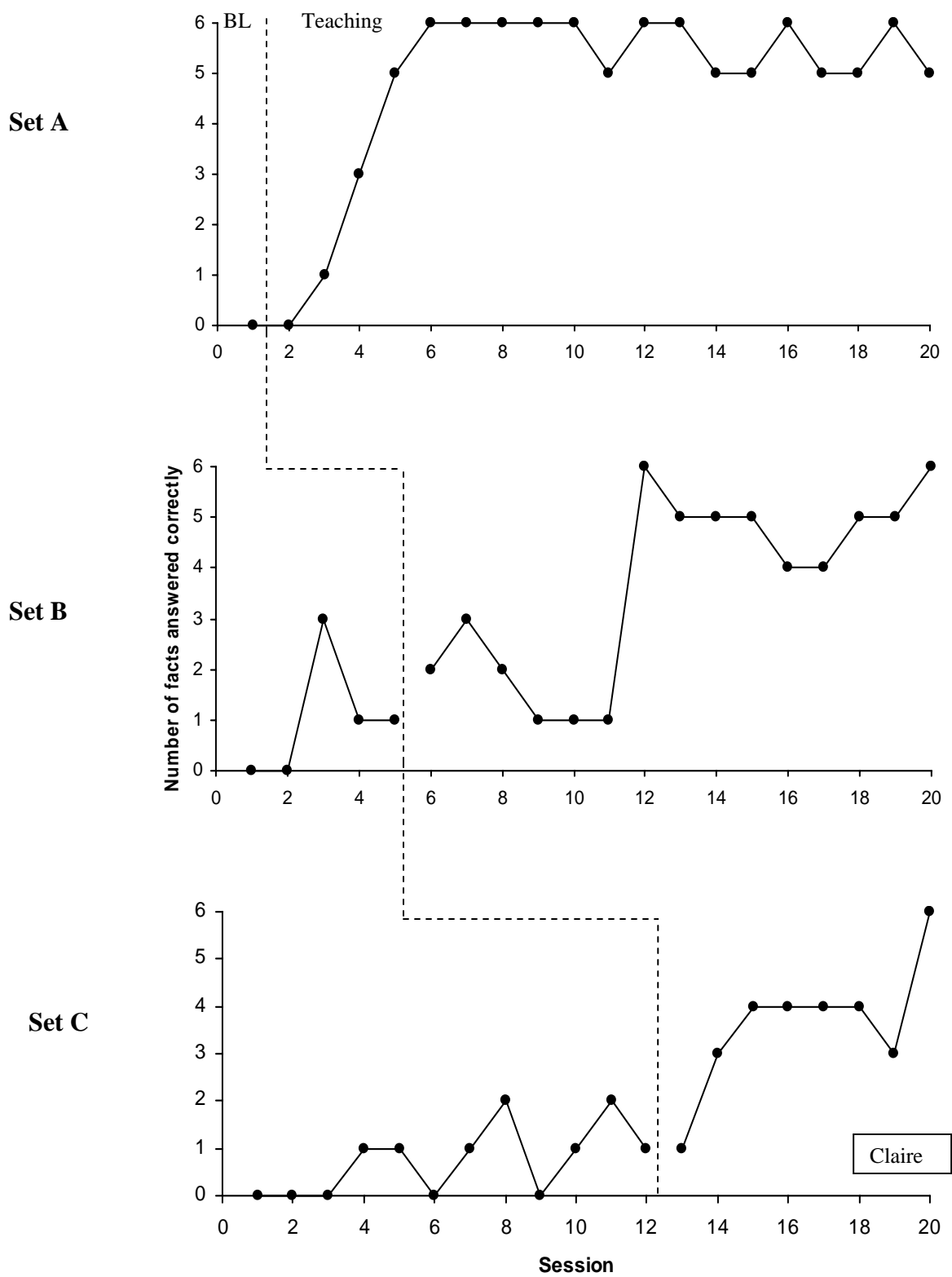
All participants' data are depicted in Figures 2, 3, and 4. There were a total of 18 facts for each participant, 6 facts per set. Once the participant could correctly answer five out of the six (four for Jack) facts during pre progress monitoring, the next set was introduced.

All participants took relatively many sessions to learn all three sets. Kate took 9 sessions to learn Set A, 11 sessions for Set B, and 5 sessions for Set C. Kate learned a total of 10 of 18 facts. Claire took 4 sessions to learn Set A, 7 for Set B, and 8 for Set C. Claire learned a total of 14 of 18 facts. Jack took 10 sessions for Set A, 9 for Set B, and 1 for Set C. Jack learned a total of 13 of 18 facts.

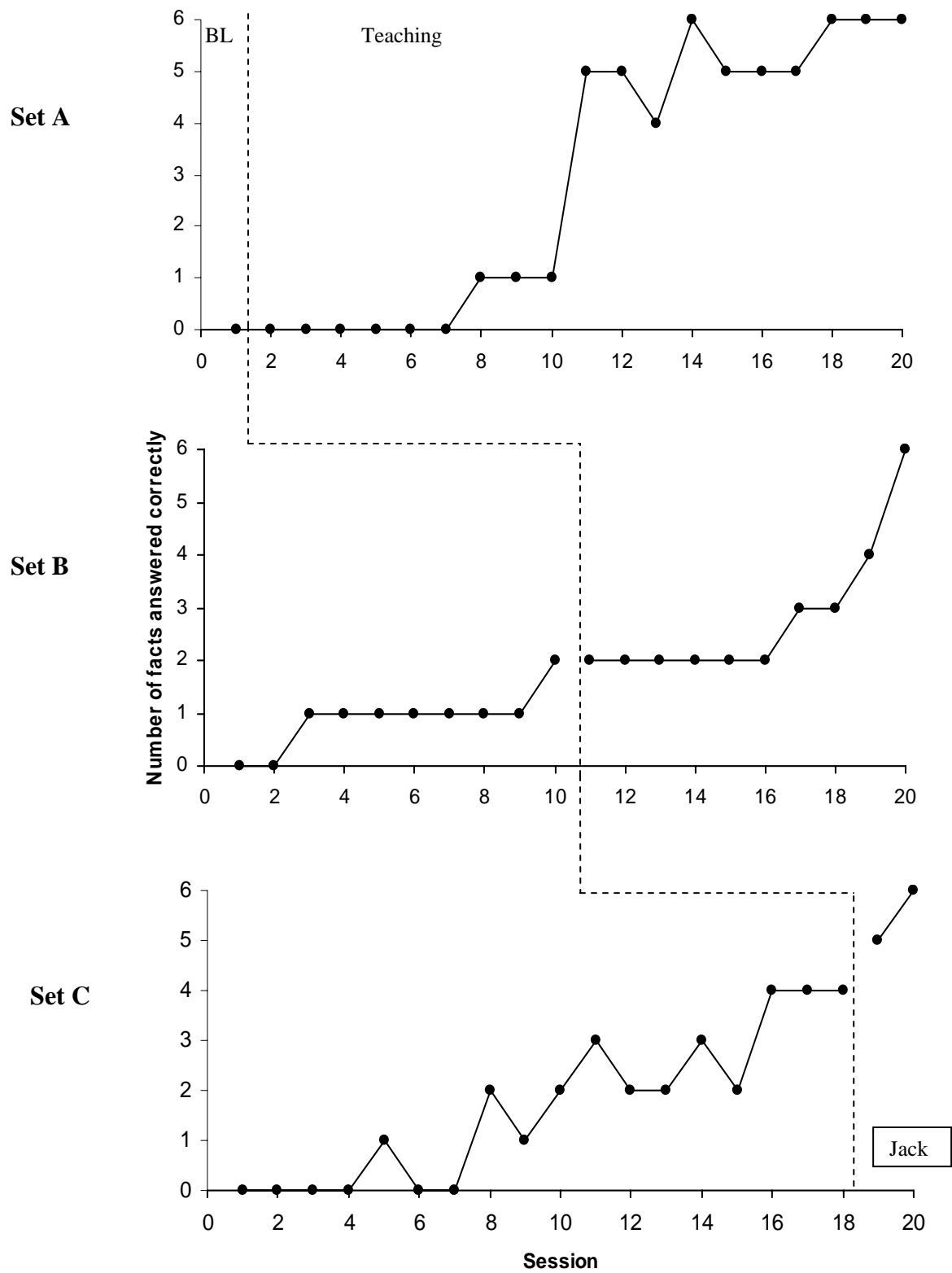
All participants continued to correctly answer previously learned facts during progress monitoring.



**Figure 2** Kate's number of facts answered correctly in Experiment II. BL represents baseline sessions.



**Figure 3** Claire's number of facts answered correctly in Experiment II. BL represents baseline sessions.



**Figure 4** Jack's number of facts answered correctly in Experiment II. BL represents baseline sessions.

## Discussion

According to some scholars mathematics instruction in the United States is lacking. Basic mathematical skills are building blocks for later achievement in mathematics and other content areas. Because most schools use a spiral curriculum addition is introduced at one point and then subtraction is introduced separately at another point. With this procedure, students are unable to see the many mathematical relationships that exist among addition and subtraction, thus possibly making it harder to learn subtraction. According to some research, students are capable of using these mathematical relationships after they are exposed to them (Baroody, 1983). The purpose of this study was to investigate the efficiency and effectiveness of teaching addition and subtraction at the same time by means of the complementary and commutative principles.

The results from Experiment 1 indicate that the procedures were too hard for these participants. Sun was able to continue with the experiment, but did not learn a substantial number of subtraction facts. She was able to learn her addition facts, but at a slower pace because she was being introduced to so many facts, addition and subtraction, at the same time. The procedure was ineffective for the other four participants.

Based on Sun's data she was able to learn her addition facts. She learned facts related to each other through the commutative principle at relatively the same pace. These findings parallel to those found by Baroody's (1983) study when he examined the use of the commutative principle and found it to be well known among most young students. In another study, Baroody (1999) found that once children were exposed to the complementary principle between addition and subtraction, learning subtraction could be easier for the child. The results from Experiment 1 do not conclusively support this

finding. Sun was starting to learn subtraction facts at a faster rate in the experimental group where the complementary principle was in use, but the rate was so slow the results were inconclusive. Baroody (1999) also suggested that learning subtraction with the use of the complementary fact was easier when the child had prior knowledge of the complementary addition facts. Sun's addition skills were low from the beginning thus delaying her ability to grasp the mathematical relationship between the addition and subtraction facts. Although, toward the end of the experiment, when Sun had mastered most of the addition facts, she was beginning to correctly answer more subtraction facts. This same problem was evident in the other four participants as well, thus hindering their ability to identify the mathematical relationships. Overall, because only one participant was able to continue with the experiment and her results do not show substantial generalization, the results for Experiment 1 are inconclusive.

The results from Experiment 2 are supportive of past findings that constant time delay is an effective method for teaching students simple addition facts. The results are clear and show that the students were able to learn addition facts using a constant time delay method. However, the rate at which the students learned the addition facts was relatively slow. This result suggests that these students may require intense sustained programmed intervention to progress even slowly. Even though the academic ability of these participants was low, constant time delay instruction was successful at teaching them simple addition facts. Thus, the results of Experiment 2 support the current literature demonstrating the effective use of a constant time delay procedure.

## Limitations and Directions for Future Research

Some limitations of the study were the number of facts in each group set for Experiment 1, a lack of cooperation among some of the participants, extraneous noises present during sessions, and experimental error. The total number of cards that were presented in each group may have been too large for the procedure. Each group contained roughly the same number of facts, but because of the use of commutative and complementary facts in the groups the number of actual fact cards was increased. The Experimental Group contained the largest number of cards because for every single fact, seven additional facts resulting from the complementary facts and the commutative facts could be used. Because the number of facts in each group was so large, the number of trials that each participant received per set during the five minute instructional session was limited. Perhaps, if the total number of facts was kept to a smaller number, the procedure may have had better results because each participant would have received more trials with each fact.

On several occasions some of the participants would not pay attention during instructional sessions and/or during progress monitoring. It would become obvious when some of the participants' were not cooperating because they answered the facts by counting from 1-10 instead of trying to answer the fact. Participants were reminded that if they did not cooperate they would not receive a reward at the end of the session. During several sessions in Experiment 1, some participants were rewarded after each group of facts taught if they cooperated. This helped to make sure that the participant was paying attention and trying to learn the facts.

Another problem arose when sessions had to be held in the teachers' lounge

because of a lack of available space within the school building. On several occasions teachers would enter the lounge and talk loudly to one another. This extraneous noise would interrupt the participant during sessions. When this occurred, the experimenter tried to keep the participant focused on the task at hand.

The lack of the participants having prior knowledge of the addition facts hindered the procedure. The participants for the study should have been chosen with stricter criteria. According to Baroody (1999), it is easier for a student to learn a subtraction fact when he/she has prior knowledge of the complementary addition fact. All participants in this study had poor addition skills. Choosing participants with higher addition skills may have resulted in more conclusive results.

Experimental error also occurred. One error was the result of the thin fact cards. It became apparent after the first session that some of the participants could see through the cards to the answer printed on the back. To correct for this, the experimenter would hold all cards up together to make it impossible to see through the cards. The data were not used for sessions when this occurred. Other errors occurred when the experimenter mixed up the group of facts. The groups were resorted and the data for that session was thrown out due to the error.

Future research should focus on new procedures for introducing the basic building blocks of mathematics. Today's current method in the most schools, a spiral curriculum, hinders a student's ability to effectively learn subtraction. This is apparent because currently most children suffer when subtraction is introduced to them by itself. Research should focus on a method that would allow addition and subtraction to be taught together to allow the students to see the mathematical relationships that exist. This may make



learning mathematics an easier task that is more efficient for students.

Future research should also focus on different areas in which using a constant time delay procedure may be beneficial. It has already been proven to be effective for many areas of concern and thus could be effective for many others.

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## Vita

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